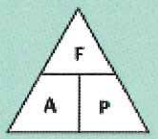
Compound measures

Yr9 Summer

Force Area Pressure



$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$

$\text{Area} = \frac{\text{Force}}{\text{Pressure}}$

Force = Area x Pressure

Examples

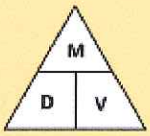
A force of 20N acted over an area of $2m^2$. What is the pressure?

$\text{Pressure} = \frac{\text{force}}{\text{Area}} = \frac{20N}{2m^2} = 10N/cm^2$

What is the force exerted on an area of $10m^2$ that is under a pressure of $2.3N/m^2$?

Force = Area x Pressure = $10m^2 \times 2.3N/m^2 = 23N$

Mass Density Volume



$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

Mass = Density x Volume

Examples

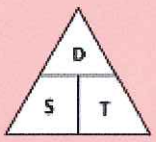
A piece of metal weighing 30g has a volume of $4cm^3$. What is it's density?

$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{30g}{4cm^3} = 7.5g/cm^3$

What is the mass of a piece of rock which has a volume of $34cm^3$ and a density of $2.25g/cm^3$?

Mass = volume x density = $34cm^3 \times 2.25g/cm^3 = 76.5g$

Speed Distance Time



$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

Distance = Speed x Time

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Examples

What is the average speed of a car that travels 400km in 5 hours?

$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{400km}{5} = 80km/h$

What is the distance covered by a train that travels at an average speed of 150mph for three and a half hours?

Distance = speed x time = $150 \times 3.5 = 525$miles

Mathematics Knowledge Organiser

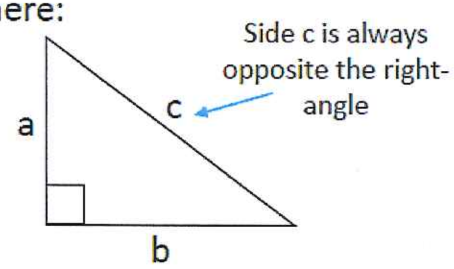
Pythagoras' Theorem and Trigonometry

Pythagoras' Theorem:
This theorem can be used to calculate the length of sides on right-angled triangles.

The theorem is:

$a^2 + b^2 = c^2$

Where:



When calculating the length of side c, use $a^2 + b^2 = c^2$

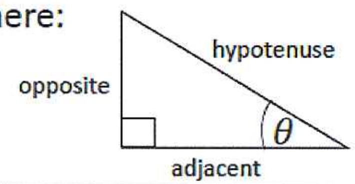
When calculating the length of a shorter side (a or b) use $c^2 - b^2 = a^2$

Trigonometry:
Trigonometry is concerned with the calculation of the length of sides and angles in triangles.

The following trigonometry functions or ratios apply to right-angles triangles:-

$\sin\theta = \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{adjacent}{hypotenuse}$ $\tan\theta = \frac{opposite}{adjacent}$

Where:



SOH-CAH-TOA!

To calculate angles, the trigonometry function needs to change sides – to do this, the inverse of the function must be used:-
$\sin^{-1}$; $\cos^{-1}$; $\tan^{-1}$

Linked Prior Topics
To calculate the square and square roots of numbers, solving equations, rearranging formulae.

Vocabulary
Pythagoras' Theorem, opposite, adjacent, hypotenuse, sine, cosine, tangent, trigonometry, trigonometric function, trigonometric ratio, inverse

Linked Future Topics
Sine rule, cosine rule, area of a triangle using trigonometry, bearings, co-ordinate geometry, angles of depression, angles of elevation, sine/cos/tan graphs

Mathematics Knowledge Organiser

Solving Quadratic Equations

Content

When we solve quadratic equations, there will be two solutions. There are 3 algebraic ways to solve quadratic equations.
You can also solve quadratic equations using a graph, find the points where the graph crosses the x-axis, these are your solutions.

Method 1 – Factorisation
First we factorise the quadratic into 2 brackets. As at least one of these brackets must equal 0. We then have 2 linear equations to solve.

Example
Solve $x^2 - 2x - 15 = 0$

Factorise into 2 brackets

$$(x+3)(x-5) = 0$$

Either

$$(x+3) = 0 \text{ or } (x-5) = 0$$

Therefore

$$x = -3 \text{ or } x = 5$$

Method 2 – Completing the Square
This method can be used when we can't easily factorise the quadratic and usually has surds in the solutions.

Example
Solve $x^2 - 4x - 3 = 0$

Rearrange so that the unknowns are on one side

$$x^2 - 4x = 3$$

Halve the coefficient of x. This number must be put into a bracket, along with x, and squared. We then subtract the square of this number

$$(x-2)^2 - 4 = 3$$

Solve the equation

$$(x-2)^2 = 7$$

$$x - 2 = \pm\sqrt{7}$$

$$x = 2 + \sqrt{7} \text{ or } x = 2 - \sqrt{7}$$

Remember that the square root of a number can be either positive or negative.

Method 3 – The Quadratic Formula
This method is also used when we cannot easily factorise the quadratic. This will be on a calculator paper and the answers will generally be decimals.

Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For quadratic equations of the form $ax^2 + bx + c = 0$

Example
Solve $2x^2 + 11x - 5 = 0$. Give your answer to 2 decimal places.

$$a = 2, b = 11, c = -5$$

Substitute these into the quadratic formula, use brackets for negative numbers.

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

Put this into the calculator, first with a + and then with a – to find your two solutions.

$$x = -5.92 \text{ or } x = 0.42$$

Linked Prior Topics
Factorising quadratic equations, solving equations, substituting into formulae

Vocabulary
Equation, quadratic, factorise, formula, substitute, coefficient

Linked Future Topics
Problem solving, sketching graphs

Mathematics Knowledge Organiser

Standard Form

- A number is converted into **standard form** when the number is very large or very small, this mainly used in science and astronomy.
- The format of a number in standard form consists of a number between 1 and 10 but **cannot be 10,** multiplied by a power of 10.

$$\boxed{(1 \leq x < 10) \times 10^n}$$

- Converting a **very small number into standard form:** Size of a bacteria is 0.00000037

  $0.00000037 = 3.7 \times 10^{-7}$

- Converting a **very large number into standard form:** Distance from Earth to the sun is 147100 million metres

  $147\,100\,000\,000 = 1.471 \times 10^{11}$

- Converting into a **small ordinary number**

  $2.4 \times 10^{-6} = 0.0000024$

- Converting into a **large ordinary number**

  $5.67 \times 10^{9} = 5\,670\,000\,000$

Common mistakes:

- When not in standard form but in the same format as the number is not between $1 \leq x < 10$

  (too big) $76.18 \times 10^6 = 7.618 \times 10^7$ and (too small) $0.12 \times 10^{-6} = 1.2 \times 10^{-7}$

When the **number is getting smaller the power gets bigger**, and when the **number gets bigger the power gets smaller.**

- When **adding or subtracting** numbers in standard form the numbers must be converted into the ordinary numbers

$$(2.3 \times 10^4) + (6.4 \times 10^3) =$$
$$23000 + 6400 =$$
$$29400 = 2.94 \times 10^4$$

- When **multiplying** numbers in standard form the format stays the same. We can use **index laws** to help us.

$$(1.5 \times 10^3) \times (3 \times 10^5) =$$
$$4.5 \times 10^{3+5} =$$
$$4.5 \times 10^8$$

1. Multiply the numbers together e.g. $1.5 \times 3 = 4.5$
2. Multiply the powers of ten together e.g. $10^3 \times 10^5 = 10^8$

- When **dividing** numbers in standard form the format stays the same. We can use **index laws** to help us.

1. Dividing the numbers e.g. $2.5 \div 5 = 0.5$
2. Dividing the powers of ten e.g. $10^{11} \div 10^{13} = 10^{-2}$

$$(2.5 \times 10^{11}) \div (5 \times 10^{13}) =$$
$$0.5 \times 10^{-2} =$$
$$5 \times 10^{-3}$$

This is not complete standard form, we multiply 0.5 by 10^1 and therefore the power reduces by 1.

Using a calculator

When inputting a very large or small ordinary number in the calculator, it will automatically convert to standard form. When inputting the number as standard form it will generally leave as standard form. You can use the button $\times 10^x$ or you can use × 1 0 $x^{\blacksquare}$.

Linked Prior Topics

- Indices
- Inequalities
- Rounding and Accuracy
- Adding, subtracting, multiplying, dividing
- Negative numbers
- Index Laws

Vocabulary

- Standard form
- Ordinary number
- Power
- Index Laws
- Convert
- Ordinary number
- Adding, subtracting
- Multiplying, dividing

Linked Future Topics

- Speed, distance and time
- Mass, density, volume
- Accuracy in answers